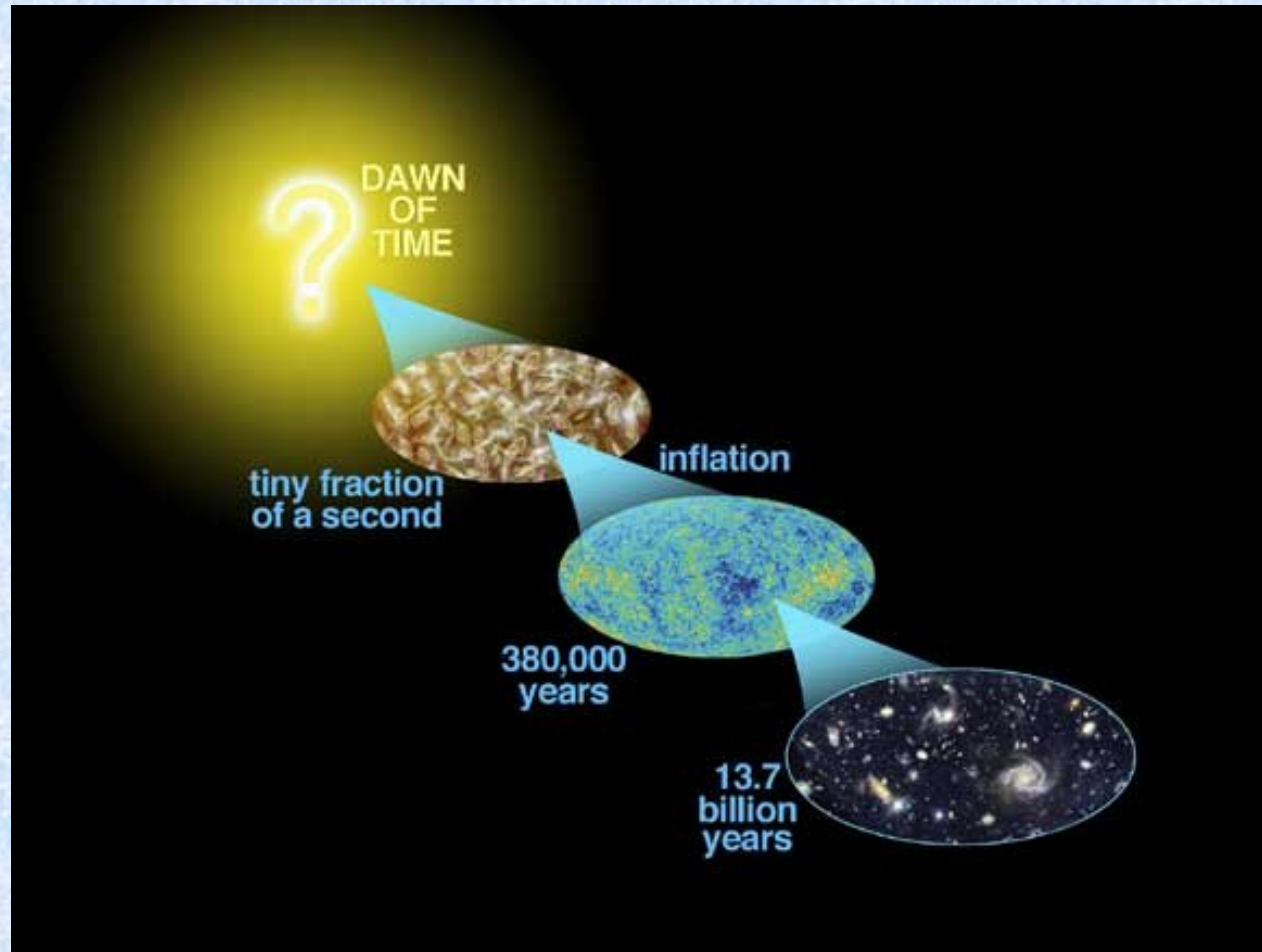


Physics 133: Extragalactic Astronomy and Cosmology



Lecture 3; January 13 2014

Previously:

- Empirical foundations of the Big Bang theory:
 - On scales larger than a 100 Mpc the Universe is isotropic. Since there is no special place in the universe (Copernican Principle) the Universe is also homogeneous.
 - The spectra of distant galaxies appear “redshifted” as if the distance between us and them was increasing with time. In the Big Bang theory this is interpreted as due to the expansion of the Universe. The timescale for expansion (i.e. the time since the Big Bang) is consistent with the age of the oldest objects known.

Previously:

- Empirical foundations of the Big Bang theory:
 - The Universe is filled with an almost perfectly isotropic blackbody radiation at a temperature of 2.7 K. This is interpreted in the Big Bang theory as the remnant of an hot state when radiation and matter were in thermal equilibrium.
 - The Universe is filled with matter (not antimatter) and a variety of particles. Baryonic matter is found to have a very regular chemical composition, mostly H, He and tiny amounts of heavier matter. *Today: only a minor fraction of particles are of known form, most of the mass of the universe is in the form of dark matter and dark energy.*

Outline:

- Types of matter/energy in the Universe
- Theoretical foundations of the Big Bang theory:
 - Gravity is the dominant force on the scale of the Universe
 - Gravity and space. Equivalence principle. Geodesics. Foundations of general relativity.
 - Non Euclidian geometries. Isotropic and homogenous surfaces.

Matter density of the Universe

- Types of matter/energy ($E=mc^2$) that we encountered so far:
 1. Radiation
 2. Neutrinos
 3. Baryons
 4. Dark matter
 5. Dark energy
- How much are they?

Matter density of the Universe.

1: Radiation

- Blackbody:
$$\rho_{\text{rad}} = 4 \sigma T^4 / c^3$$
- Where c is the speed of light, T is the temperature, σ is the Stefan-Boltzmann constant), $5.67\text{e-}8 \text{ W m}^{-2} \text{ K}^{-4}$
- So $\rho_{\text{rad}} = 4.6\text{e-}31 (T/2.725\text{K})^4 \text{ kg/m}^3$



Matter density of the Universe.

1: Radiation in critical units

- It is convenient to write this down in terms of the critical density, the amount of energy/matter needed to “close” the universe
- Defined as:
 - $\rho_{\text{crit}} = 3H_0^2/8\pi G$
 - $= 9.5e-27 \text{ kg/m}^3$
 - $= 6 \text{ H atoms} / \text{m}^3$
- The density of radiation is $4.8e-5 \rho_{\text{crit}}$
- This can be written as $\Omega_{\text{rad}} \sim 5e-5 \Omega_{\text{crit}}$



Matter density of the Universe.

2: Neutrinos

- Limits on neutrino mass density come from:
 - Oscillations (lower limit; superkamiokande)
 - large scale structures (upper limits; CMB+2dF; Giusarma et al. 2013 <0.35eV)
- In critical units:
- $\Omega_{\text{neu}} = m_{\text{neu}} c^2 h^2 / (93 \text{ eV})$
- $= 1e-3 - 8e-3$



Matter density of the Universe.

3: Baryons

- People have counted the amount of mass in visible baryons.
- Baryonic inventory (total= 0.045 ± 0.003 from nucleosynthesis and CMB):
 - Stars $\Omega_* = 0.0024 \pm 0.0007$ (more mass in neutrinos than in stars!)
 - Planets $\Omega_{\text{planet}} \sim 10^{-6}$
 - Warm intergalactic gas 0.040 ± 0.003
- Most of baryons are in intergalactic medium, filaments in the cosmic web,



Matter density of the Universe.

4: Dark matter

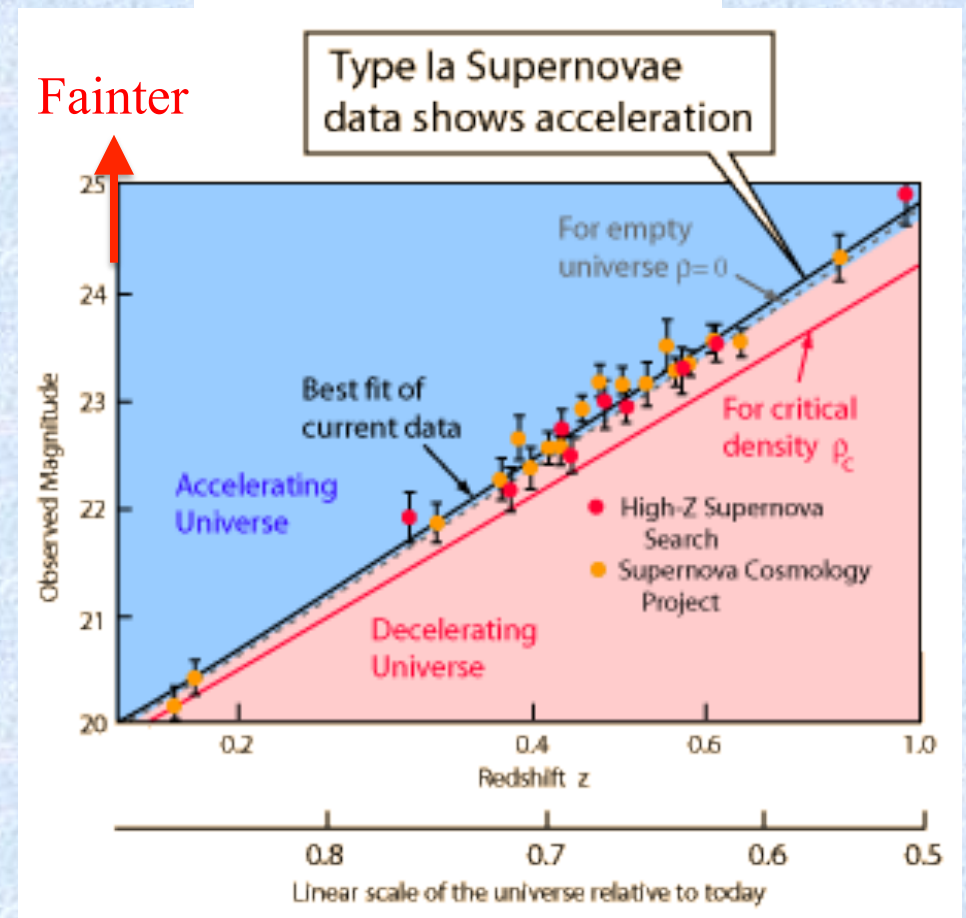
- Dark matter is harder to count, because we can only “see” it via its gravitational effects
- One way to count it is for example is to measure the dark matter to baryon ratio in clusters
- Assume that this number is representative of the Universe because the collapsed volume is large
- Take the fraction of baryons (from BBN) and multiply
- This and other methods give $\Omega_{\text{dm}}=0.23$
- The total amount of matter is given by: $\Omega_{\text{m}}=\Omega_{\text{dm}}+\Omega_{\text{b}}=0.27$



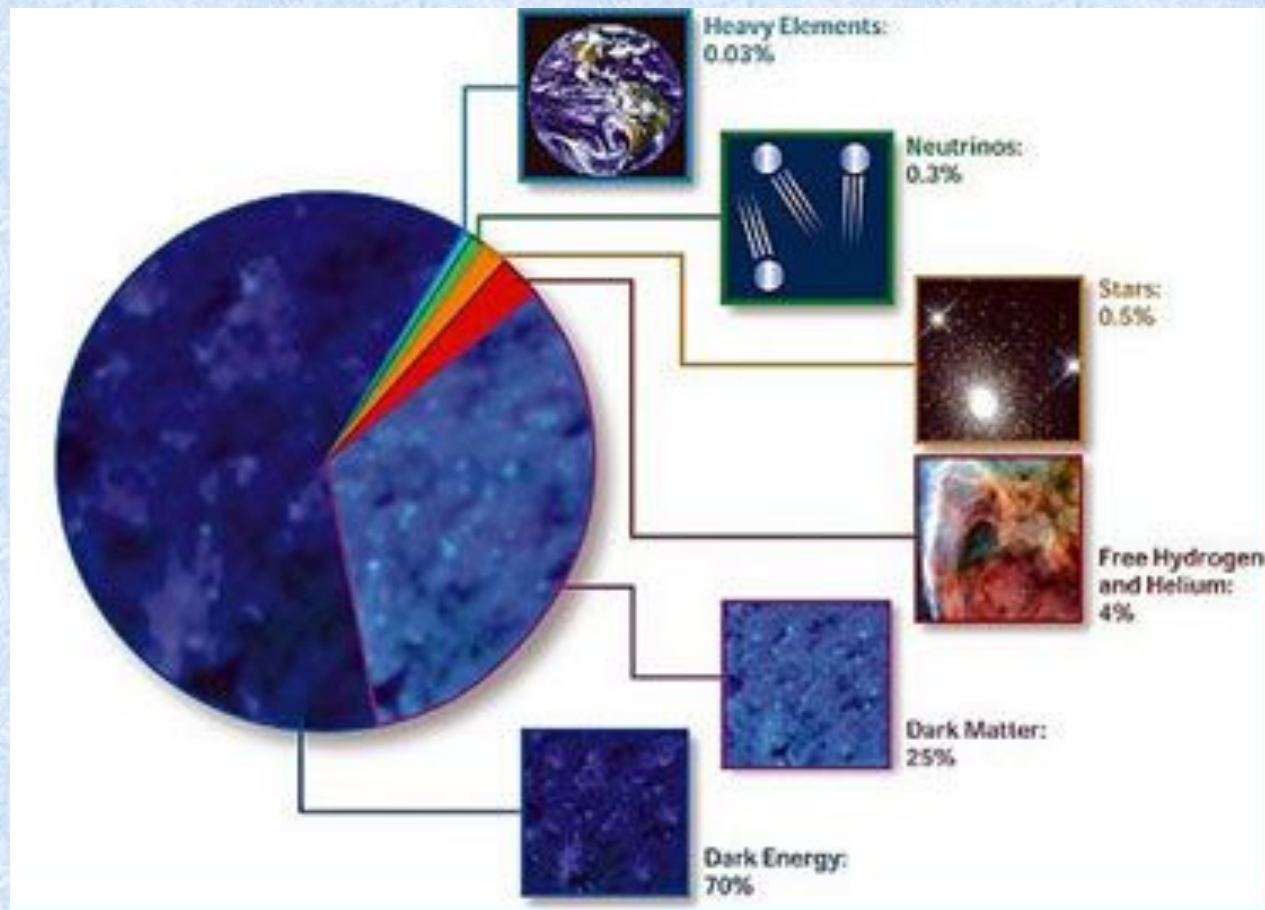
Matter density of the Universe.

5: Dark energy (or Λ)

- As we will see most of the energy in the universe appears to be of a mysterious form called dark energy
- Dark energy repels instead of attracting, and therefore causes the expansion of the universe to accelerate.
- One form of dark energy is the cosmological constant (Λ), introduced by Einstein a long time ago, and this is a purely geometrical term... We will explain this later
- According to current measurements $\Omega_{de} \sim 0.72$ or $\Omega_{\Lambda} \sim 0.72$.



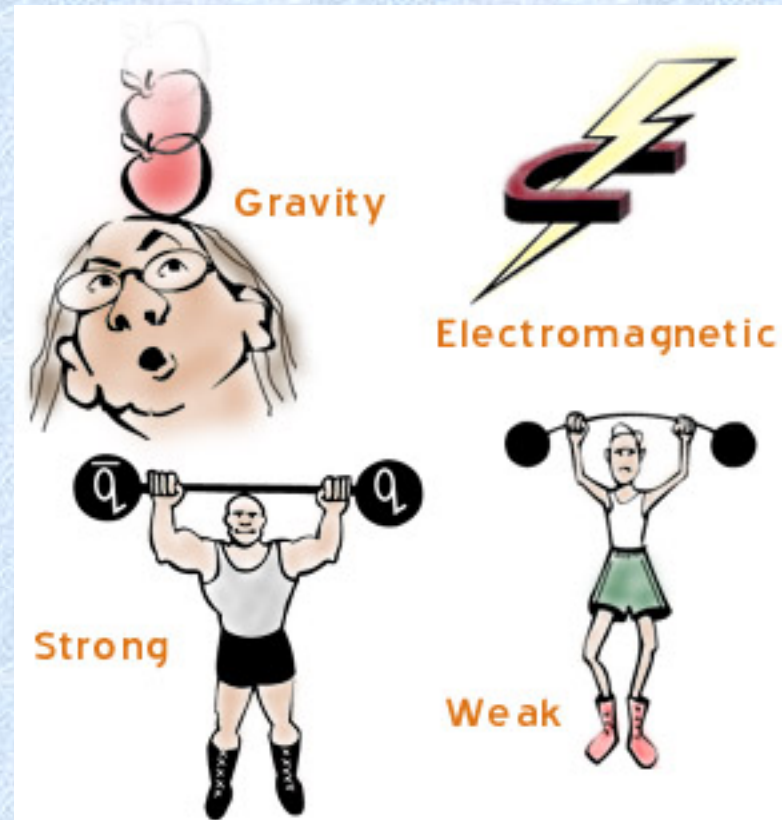
Matter density of the Universe. Summary



Total matter density $\Omega_{\text{rad}} + \Omega_{\text{neu}} + \Omega_{\text{m}} + \Omega_{\Lambda} = 1.0$ (within $<1\%$)

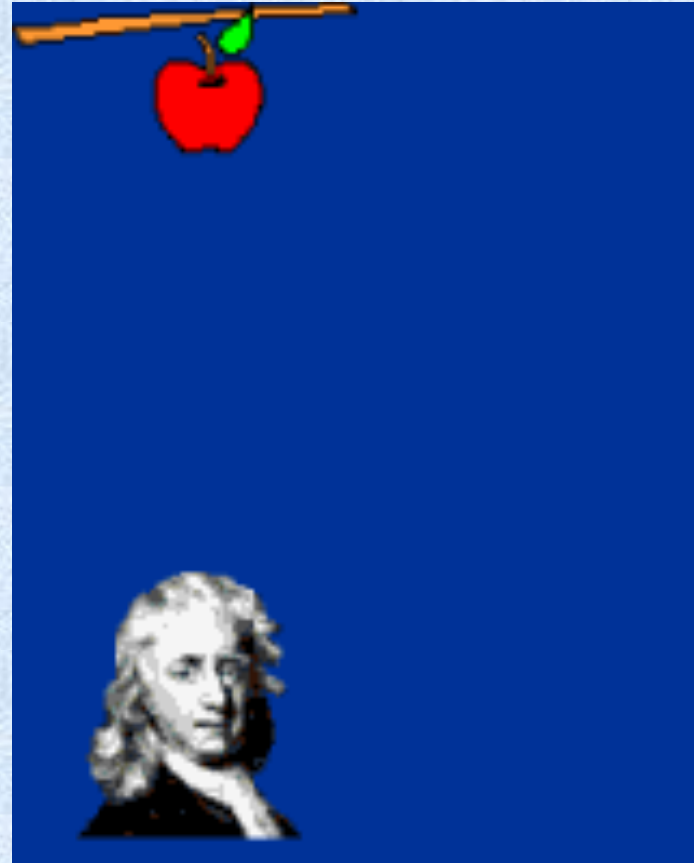
The four forces

- In our current understanding of physics all interactions are due to 4 forces:
 1. Gravity
 2. Electromagnetic
 3. Strong Interactions
 4. Weak Interactions



Gravity

- Main properties:
 - Long range
 - Only attractive
 - Very weak force
 - Consider the ratio of the gravitational and electric attraction between a proton and an electron:
 - $FG = G m_p m_e / R^2$
 - $FEM = k Q^2 / R^2$
 - $FEM / FG = 10^{39} !$
 - Exchange boson: graviton
- Example of systems dominated by gravity?
 - Universe
 - Black hole
 - Solar system



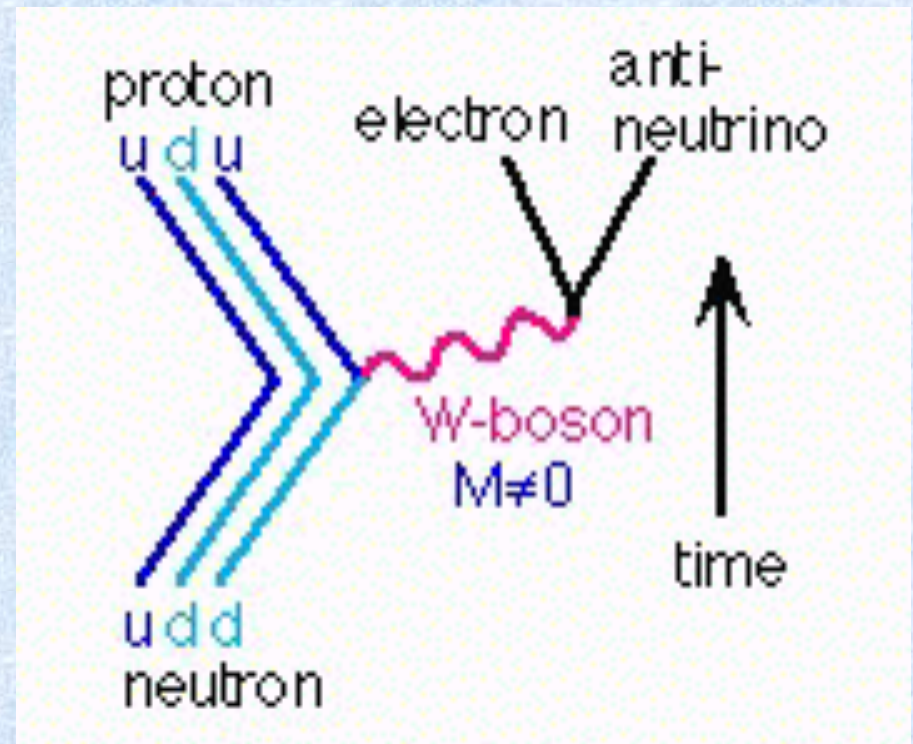
Electromagnetic force

- Main properties:
 - Long range
 - Attractive and repulsive
 - Much stronger than gravity but effectively “shielded over long distances”
 - Exchange Boson: photon
 - NB: E&M is unified description of electricity and magnetism
- Examples of systems:
 - Atoms (electrons and nuclei)
 - Electromagnetic waves: light, cell phone...



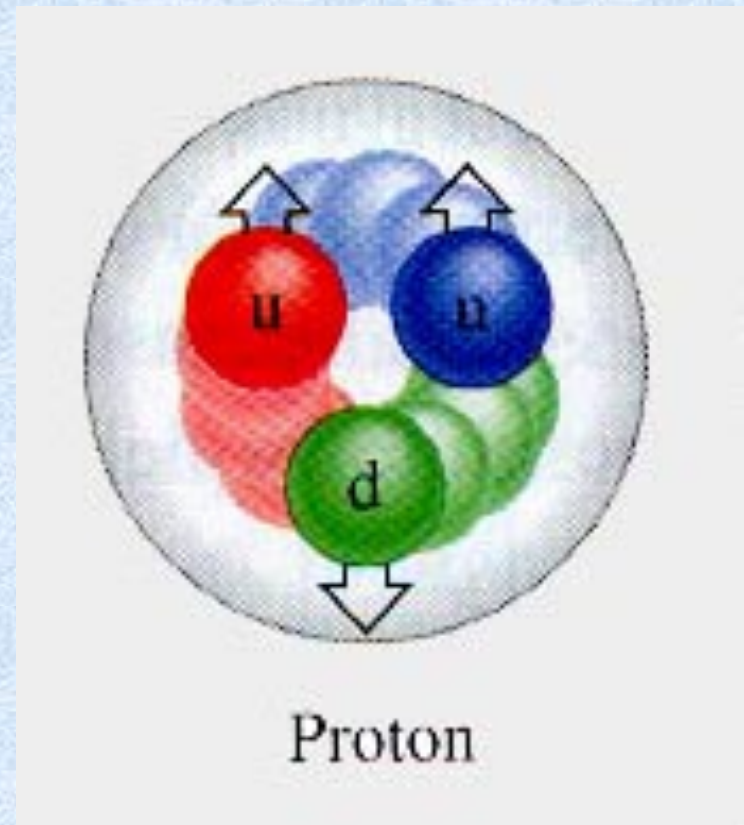
Weak force

- Main properties:
 - Short range
 - Responsible for change of flavor of quarks (e.g. neutron decaying into proton)
 - VERY WEAK!!
 - Exchange Boson: W^{+-} , Z_0
- Examples of systems:
 - Neutrino interactions
 - Beta decays



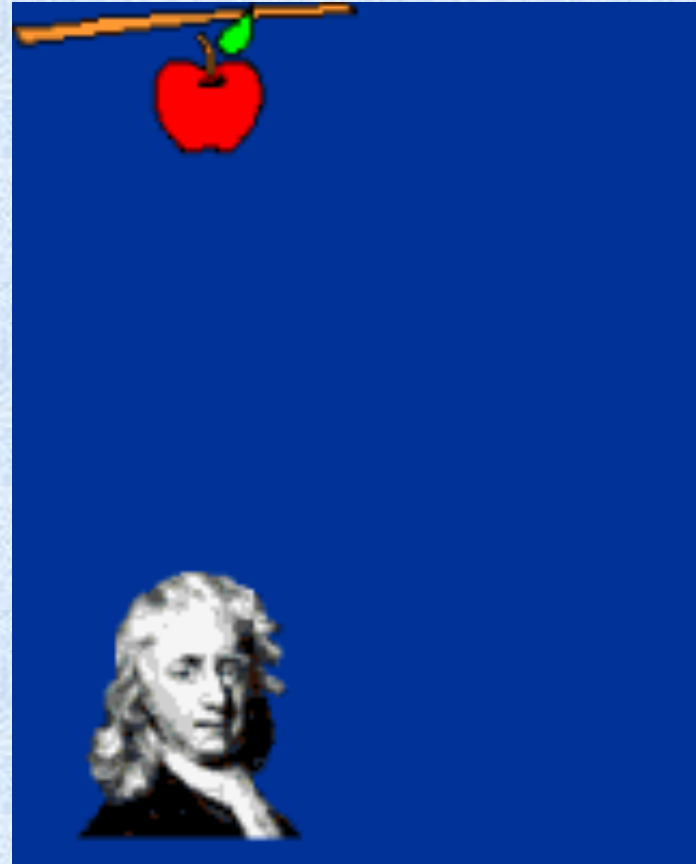
Strong force

- Main properties:
 - Short range
 - Holds quarks (and nuclei) together
 - VERY STRONG!!! (keeps protons together even though they have the same electric charge)
 - Exchange Boson: gluons
- Examples of systems:
 - Nuclei of atoms



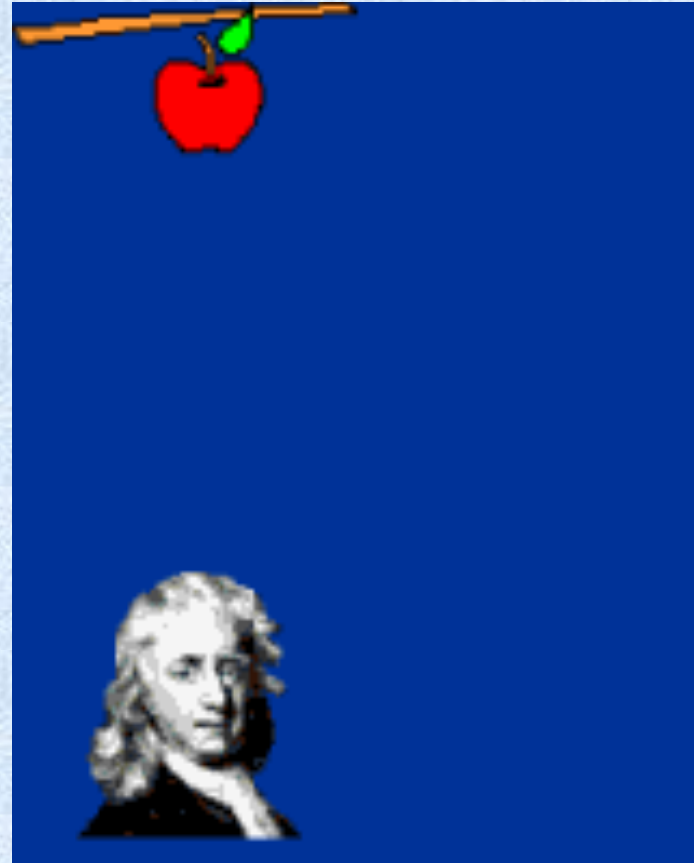
Gravity is the dominant force

- Weak and strong interactions are short range
- There are negative and positive charges so that charges and currents tend to shield each other and neutralize each other over long distances.
- Gravity, is the winner!

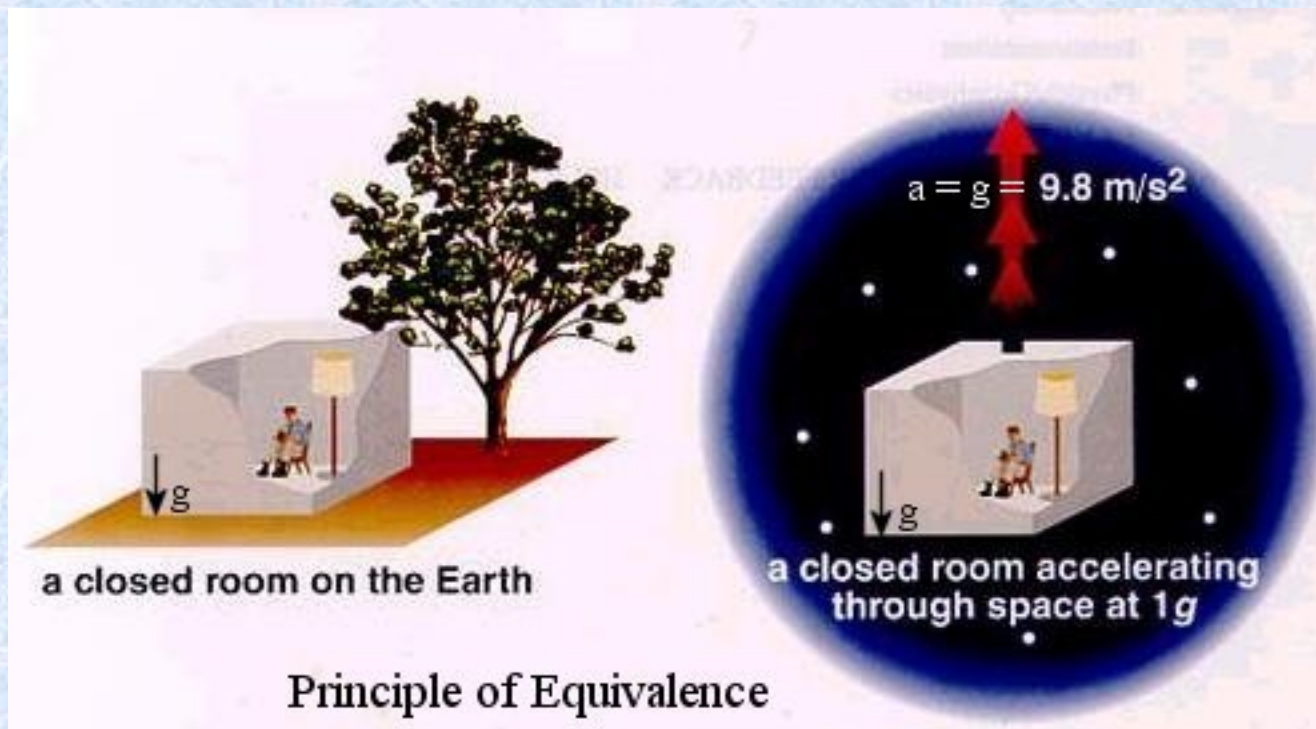


Newton's gravity

1. Space is Euclidian. There are bodies and gravity
2. $F = -G M_g m_g / R^2$
3. $F = m_i a$
4. $a = G M_g / R^2 (m_g/m_i)$
5. Coincidence: inertial and gravitational mass are experimentally the same to within 1 part in 10^{12}
6. All objects accelerate in the same way on the Earth's surface (Galileo's experiment)

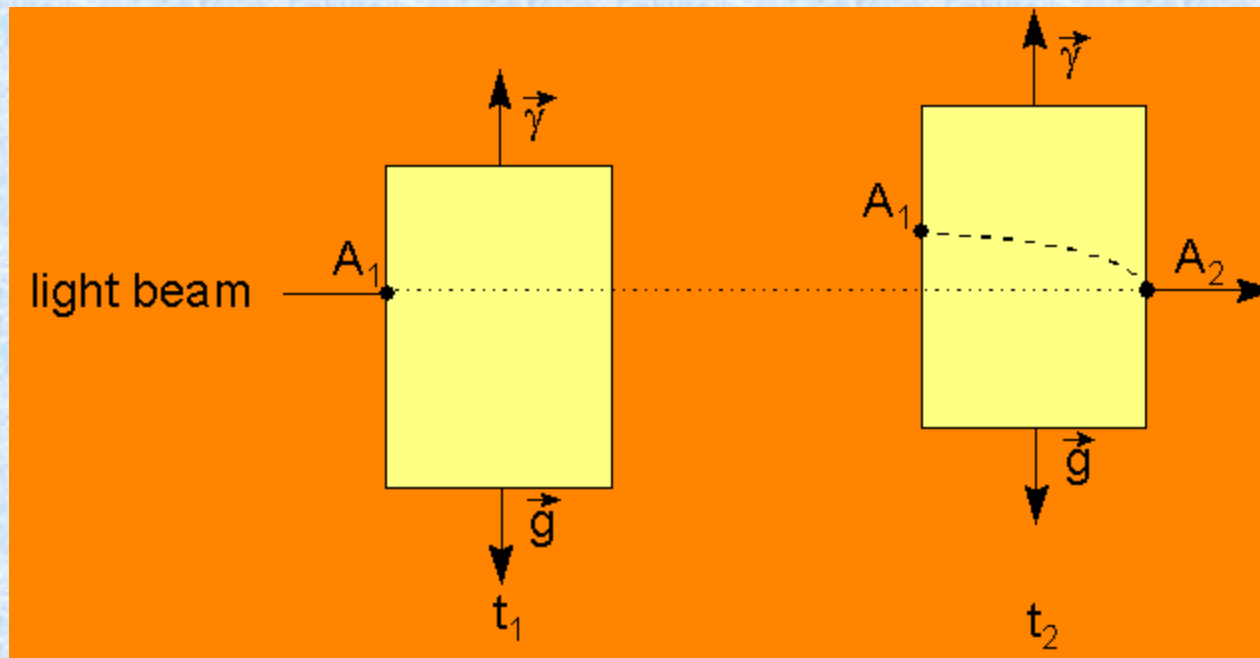


Einstein's gravity. Equivalence principle



Equivalence of inertial and gravitational mass is exact!

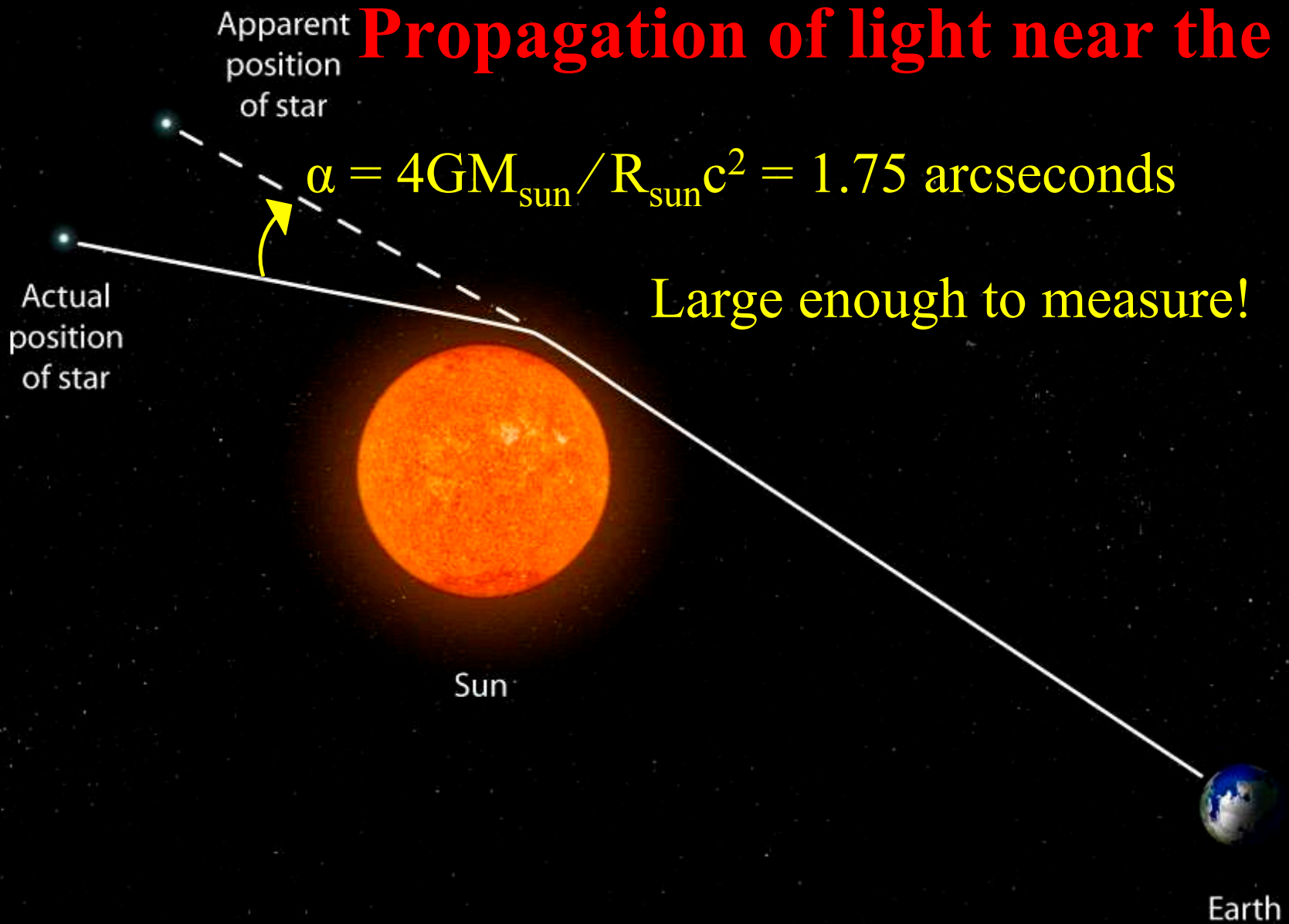
Einstein's gravity. Propagation of light.



If the elevator is accelerating (or in a gravitational field) the beam of light appears to bend as it propagates through the elevator.

Gravity affects photons even though they have no mass!

Propagation of light near the sun





LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less
Agog Over Results of Eclipse
Observations.

EINSTEIN THEORY TRIUMPHS

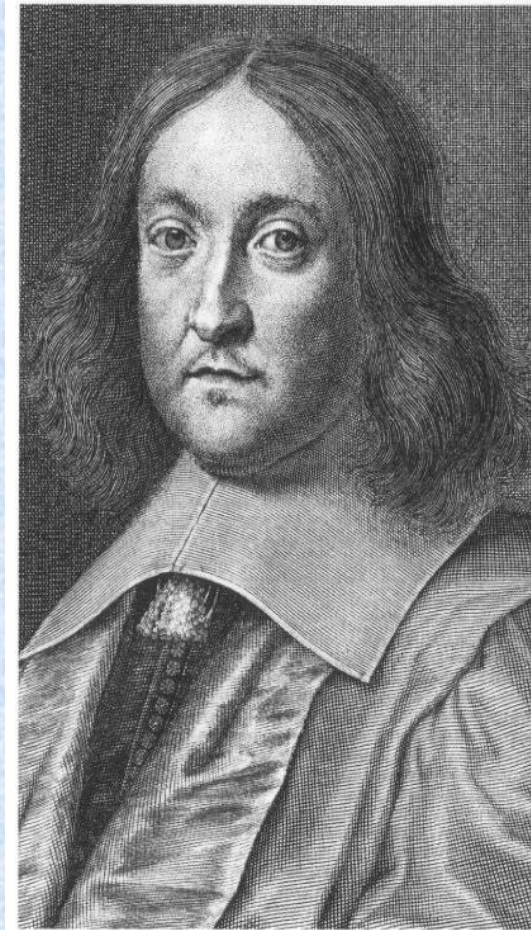
Stars Not Where They Seemed
or Were Calculated to be,
but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could
Comprehend It, Said Einstein When
His Darling Publishers Accepted It.

Einstein's gravity. Fermat's principle.

1. A fundamental principle of optics is that light travel along the line(s) that minimize travel time (later on we'll see that it travels along extrema of the time delay surface)
2. In flat Euclidian space these are straight lines
3. What about the elevator? And the case of light bending near the sun?



Einstein's gravity.

The bottom line.

1. Since light does not travel along straight lines, space is not flat!
2. There is no gravity. Space-time is a non-Euclidian four-dimensional space, and objects move along geodesics in this curved space.
3. Energy and mass are equivalent ($E=mc^2$). Energy and mass curve space time.
4. Einstein's equation connects matter to curvature of space time

Newton vs Einstein:

- **Newton:** Gravitational mass tells gravity how to exert a force. Force tells inertial mass how to accelerate. Inertial and gravitational mass are the same just by coincidence.
- **Einstein:** Mass-energy tells space-time how to curve. Curved space time tells mass-energy how to move.

[Blackboard]

Non-Euclidean geometry.

Examples in 2D. Plane.

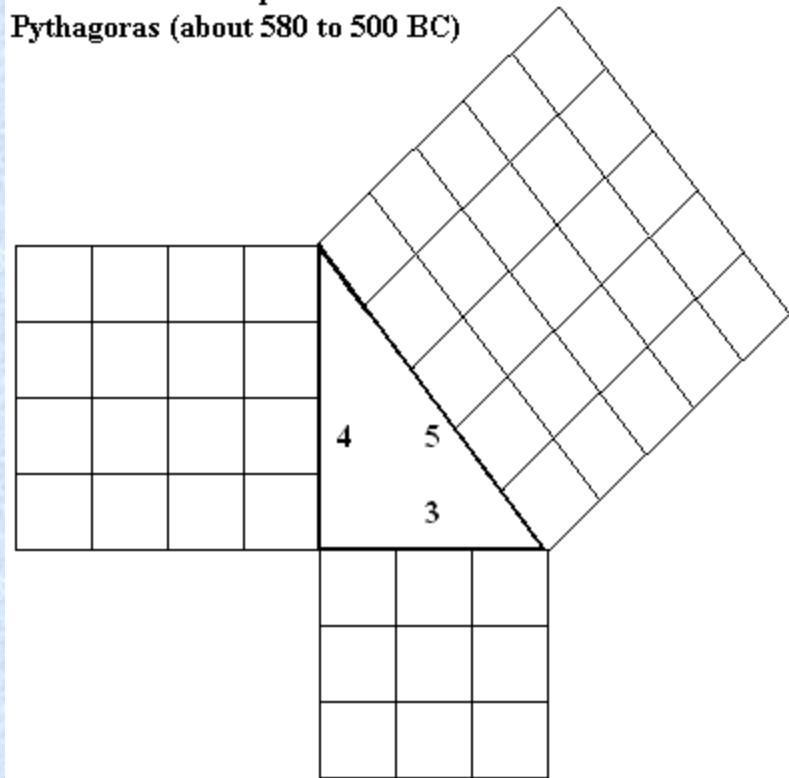
1. In Euclidean geometry.
The angles of a triangle add up to π
2. The distance between two points is given by:
 $ds^2 = dx^2 + dy^2$
3. In polar coordinates:
 $ds^2 = dr^2 + r^2 d\theta^2$
4. These expressions are called the metric

3.2.1 Right-angled triangle

The theorem of Pythagoras

In a right-angled triangle, the square on the hypotenuse is the sum of the squares on the other two sides.

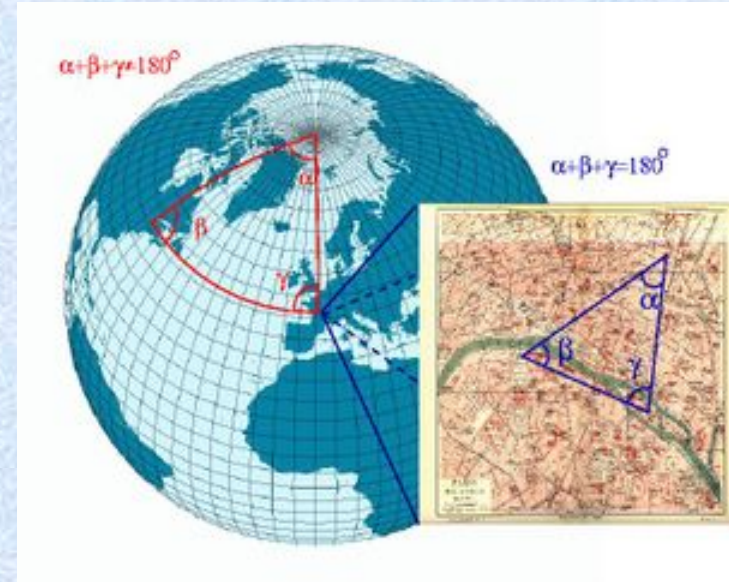
Pythagoras (about 580 to 500 BC)



Non-Euclidean geometry.

Examples in 2D. Sphere.

1. On the surface of a sphere, the angles of a triangle add up to $\pi + A/R^2$
 1. Does not depend on the location on the sphere, isotropic and homogeneous
2. The distance between two points is given by:
$$ds^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2$$
 1. The surface is finite, and there is a maximum distance
3. The surface of a sphere, as all spaces where the angles of a triangle add up to more than π , is **positively curved**



Non-Euclidean geometry.

Examples in 2D. Hyperboloid.

1. On the surface of a negatively curved surface the sum of the angles is $\pi - A/R^2$
2. The distance between two points is given by:
$$ds^2 = dr^2 + R^2 \sinh^2(r/R) d\theta^2$$
3. The surface is infinite, and there is no maximum distance

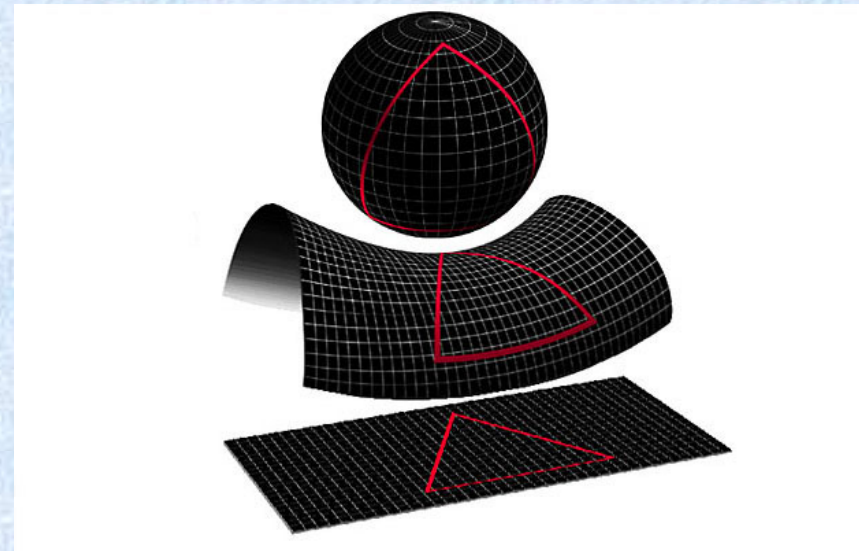


Non-Euclidean geometry.

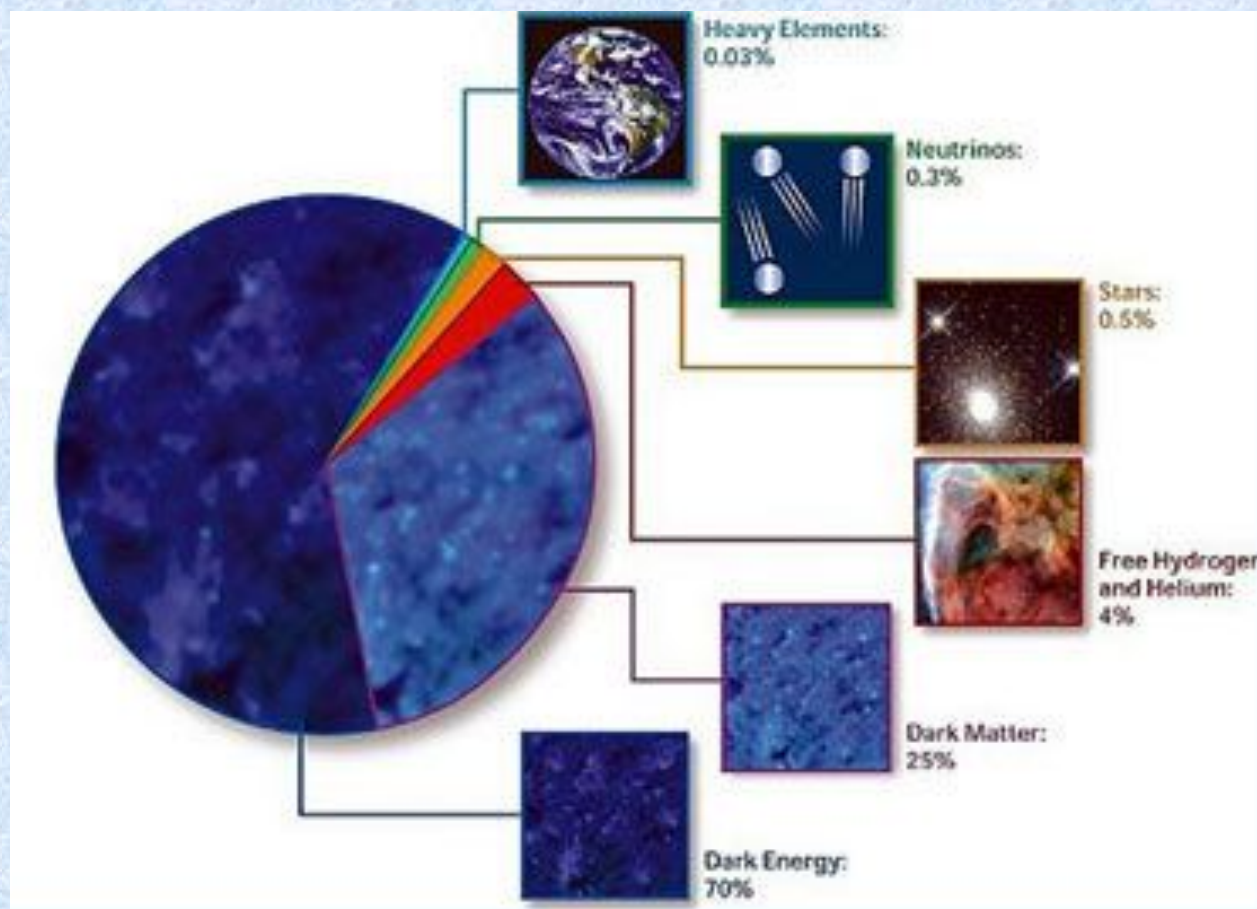
3D isotropic surfaces.

1. Curvature is a local property. Isotropic and homogenous spaces need to have a constant curvature, which can be zero (flat), negative, or positive.
2. All the properties of the surface are described by the sign $k=-1,0,+1$ and radius of curvature R .
3. The metric in spherical coordinates is given by:
$$ds^2=dr^2+S_k(r)^2 d\Omega^2$$
4. [Blackboard]

2D analogs



Matter density of the Universe, revisited



Total matter density $\Omega_{\text{rad}} + \Omega_{\text{neu}} + \Omega_{\text{m}} + \Omega_{\Lambda} = 1.0$ (within $<1\%$)

Curvature energy density: $\Omega_{\text{k}} = 1 - \sum \Omega_{\text{mass+energy}} = 0$

→ Spacetime is almost exactly flat (Euclidean)

Theoretical foundations of the Big Bang. Summary

- The dominant force on the scale of the Universe is gravity
- Gravity is accurately described by the theory of general relativity
 - Mass-energy tells spacetime how to curve and spacetime tells mass-energy how to move
- In three dimensions there are only 3 possible isotropic and homogeneous manifolds
 - Positive curvature (hyper-sphere)
 - Negative curvature (hyper-hyperboloid)
 - Zero curvature (flat Euclidean space)

The End

See you on wednesday!